

# Duality of thermal and dynamical descriptions in particle interactions

J.Cleymans<sup>1</sup>, G.I.Lykasov<sup>2</sup>, A.N.Sissakian<sup>2</sup>, A.S.Sorin<sup>2</sup>, O.V.Teryaev<sup>2</sup>

<sup>1</sup> *UCT-CERN Research Center and Department of Physics, University of Cape Town, Rondebosch 7700, Cape, Republic of South Africa*

<sup>2</sup> *JINR, Dubna, 141980, Moscow region, Russia*

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## Abstract

We suggest a duality between the statistical and standard (dynamical) distributions of partons in the nucleons. The temperature parameter entering into the statistical form for the quark distributions is estimated. It is found that this effective temperature is practically the same for the dependence on longitudinal and transverse momenta and, in turn, it is close to the freeze-out temperature in high energy heavy-ion collisions.

*Keywords:* Statistical nucleon structure functions, deep inelastic scattering

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## 1. Introduction

As is known, the statistical (thermal) models have successfully been applied to describe ratios of hadronic yields produced in heavy-ion collisions (see, for example, [1, 2], [3]-[5] and references therein). At the same time, the source of very fast thermalization is currently unknown and alternative or complementary possibilities to explain the thermal spectra are of much interest.

Situations where statistical models were applied while the notion of statistical system was not obvious are not uncommon. In particular, the statistical model was successfully applied to analyze deep inelastic lepton-nucleon scattering (DIS) [6, 7], where the statistical form [8, 9] for distributions of unpolarized and polarized quarks and gluons (partons) in a nucleon was used.

This model may be compared with more standard parametrization of parton distributions based on the Regge theory at low  $x$  and quark counting rules at large  $x$  [10]-[12]. It is especially interesting to find a counterpart

and understand a physical meaning of the *universal temperature* introduced in the statistical model. This is the main goal of this paper. Making this comparison we suggest a concept of the *duality* between the statistical and dynamical descriptions of the DIS and strong interactions of particles and explore its consequences and implications.

## 2. Statistical and dynamical description of quark distributions

### 2.1. Quark distribution in proton: longitudinal momentum

According to many experimental data, inclusive spectra of hadrons produced in heavy-ion collisions can be fitted using a thermodynamic statistical distribution as a starting point for the system of final hadrons, see for example [1, 4]

$$f_h^A = C^A \{ \exp((\epsilon_h - \mu_h)/T) \pm 1 \}^{-1}, \quad (1)$$

where  $+$  is for fermions and  $-$  is for bosons,  $\epsilon_h$  and  $\mu_h$  are the kinetic energy and the chemical potential of the hadron  $h$ ,  $T$  is the temperature, and  $C^A$  is the normalization coefficient. For mesons simplifying this case we can assume that  $\mu_h \simeq 0$ , (in fact, it generally cannot be strictly zero [5]). At high energies, when  $\epsilon_h/T \gg 1$ , Eq.(1) is usually presented in the Boltzmann statistic form

$$f_h^A \simeq C^A \exp(-\epsilon_h/T). \quad (2)$$

A statistical approach was applied in great detail to study the quark structure of nucleons in Ref. [6], using earlier approaches as a starting point [8, 9]. The nucleon is viewed as a gas of massless partons (quarks, antiquarks, gluons) in equilibrium at a given temperature in a finite size volume. According to the thermodynamical approach, the quark distribution in a proton  $q(x)$  in the infinite momentum frame (IMF) at the input energy scale  $Q_0^2$ , which is used in the perturbative QCD [13] as the initial condition for the  $Q^2$  evolution, is fitted as a function of the light cone variable  $x$  in the following form:

$$xq(x) = \frac{B_1 X_{0q} x^{b_1}}{\exp((x - X_{0q})/\bar{x}) + 1} + \frac{B_2 X_{0q} x^{b_2}}{\exp(x/\bar{x}) + 1}, \quad (3)$$

where  $\bar{x}$  plays the role of a *temperature inside the proton* and  $X_{0q}$  is the chemical potential of the quark inside the proton. All the parameters are found

from the description of the DIS. The same form is suggested in [7] for anti-quarks  $x\bar{q}(x)$  by changing the sign for the chemical potential  $X_{0q}$ . Therefore, the proton structure function  $F_2(x, Q^2) = \sum_{flavour} x (q_f(x, Q^2) + \bar{q}_f(x, Q^2))$  has also a thermodynamical form like Eq.(3).

Note that the statistical weight of the quarks can be written in the form  $\exp((E_q - \mu)/T)$ , where  $E_q = (P \cdot p_q)/m$  is a quark energy in the nucleon rest frame,  $p_q$  and  $P$  are the four-momenta of quark and nucleon, respectively,  $m$  is the nucleon mass. Passing to the infinite momentum frame the quark distribution in a nucleon over the longitudinal momentum  $p_z$  can be represented in the following statistical form [9]:

$$q(p_z) \sim \exp \left\{ - \left( \frac{E\epsilon - Pp_z}{mT} - \frac{\mu}{T} \right) \right\} , \quad (4)$$

where  $E$ ,  $P$  and  $m$  are the energy, the momentum and the mass of the nucleon moving in the  $z$ -direction,  $\epsilon$  and  $p_z$  are the energy and momentum in the  $z$ -direction of a parton in the nucleon,  $T$  is the temperature and  $\mu$  is the chemical potential. For massless partons and  $\mu \simeq 0$  one gets

$$q(x) \sim \exp \left( - \frac{mx}{2T} \right) \equiv \exp \left( - \frac{x}{\bar{x}} \right) . \quad (5)$$

where  $x = p_z/P$  is the longitudinal fraction of the parton momentum and  $\bar{x} = 2T/m$ . This form agrees with the result obtained in [8, 9]. The inclusion of the quark transverse momentum  $k_{qt}$  results in  $E\epsilon - Pp_z \simeq m^2x(1 + k_{qt}^2/(x^2m^2))$ . It means that the relativistic invariant variable  $x$  is replaced by the variable  $x(1 + k_{qt}^2/(x^2m^2))$  [14].

The statistical parametrization of parton distributions [6, 7] may be compared with a more standard form (see, e.g., [10]) describing the experimental data on the DIS:

$$xq(x) = A_s x^{-a_s} (1-x)^{b_s} + B_v x^{a_v} (1-x)^{b_v} , \quad (6)$$

where the first term is due to the sea quark distribution which is enhanced at low  $x$ , according to the DIS experimental data, whereas the second term is due to the valence quark distribution. Note that to fit the experimental data on the DIS a more complicated form for  $xq(x)$  is also used [11, 12].

In particular, the valence part at  $x < 1/b_{qv}$  can, to a very good approximation, be written as:

$$xq_v(x) \simeq B_{qv} x^{a_v} \exp(-xb_{qv}) . \quad (7)$$

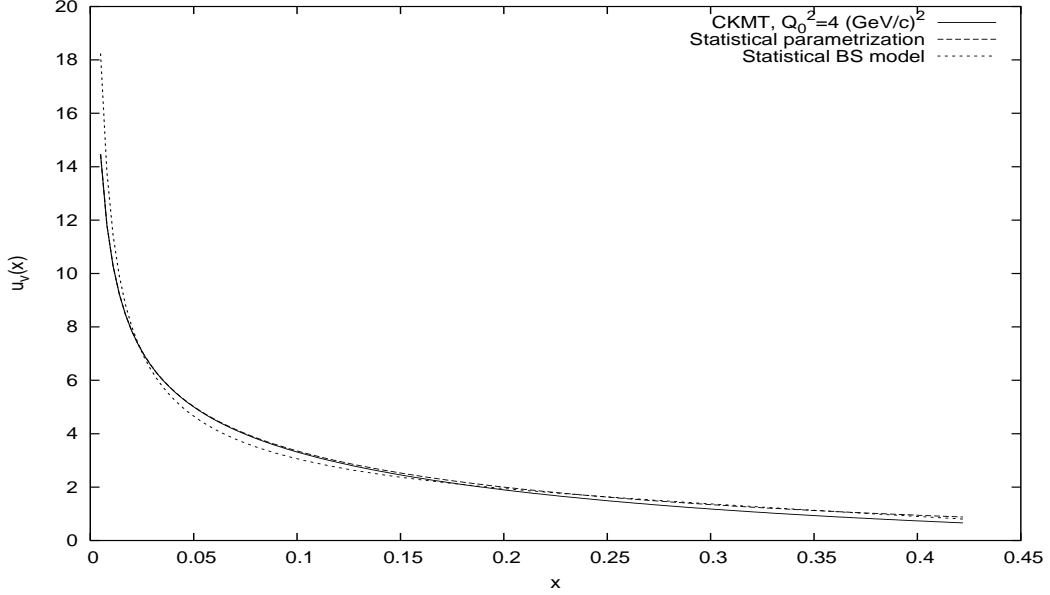


Figure 1: The valence  $u$ -quark distribution in the proton as a function of the Bjorken variable  $x$  at  $Q_0^2 = 4(\text{GeV}/c)^2$ . The solid curve is the CKMT parametrization [10], the dashed line corresponds to the statistical parametrization given by Eq.(3) the dotted curve corresponds to the BS statistical (thermal) model suggested in [6, 7].

This form is similar to Eq.(5) if  $\bar{x} = 1/b_v$ . In this case the proton temperature is  $T = \bar{x} \cdot m/2 = m/(2b_v(Q_0^2))$  for current quarks which are practically massless. Approximately one has for the valence  $u$ -quarks  $b_{u_v} \simeq 2.5 - 3$  and for  $d$ -quarks  $b_{d_v} \simeq 3.5 - 4$  at  $Q_0^2 = 2 - 4 (\text{GeV}/c)^2$  [10, 12]; therefore,  $T \simeq 120 - 150$  MeV for the massless quarks.

The similarity of the thermal form for the quark distribution given by Eq.(5) and the dynamic form for  $q_v$  (Eq.(7) maybe interpreted as *a duality of the thermal and dynamical* descriptions of the parton distribution in proton. This property is by no means surprising as both parametrization are fitted to describe the same data. In Fig.1, the valence  $u$ -quark distribution in the proton as a function of  $x$  is presented, the solid curve corresponds to the CKMT parametrization [10], see the second term of Eq.(6) at  $Q_0^2 = 4(\text{GeV}/c)^2$ ; the dotted line corresponds to the statistical (thermal) BS model [7], see the first term of Eq.(3) at the same value of  $Q_0^2$ ; the dash-dotted curve corresponds to the statistic parametrization given by Eq.(7). Figure 1 shows that all three lines are very close to each other at  $0.01 \leq x \leq 0.4$ .

Let us compare the parameters of the statistical parametrization for  $u_v(x)$

given by Eq.(7) and the BS parametrization (Eq.(3)). While the temperature has an evident dynamical counterpart  $T = m/(2b_v)$ , a similar relation for the chemical potential  $X_{0q}$  of valence quarks is not obvious. The temperature  $T = m/(2b_v)$  entering into Eq.(7) is about 120–150 MeV, whereas the model of Ref. [7] suggests that  $T$  is about 50 MeV and the chemical potential for  $u$ -quarks  $X_{0u} \simeq 216$  MeV as first found in [8, 9].

## 2.2. Quark distribution in proton: transverse momentum

The quark distribution function in a proton using the variable  $x$  and the transverse momentum  $k_t$  can be represented in the factorized form  $f_q(x, k_t) = q(x)g(k_t)$  which of course cannot be valid at all values of  $x$  [15]. This is supported by simulations within the lattice QCD [16] and by the observation of the so-called “seagull” effect [17], i.e., the weak  $x$ -dependence of the mean transverse momentum  $\langle p_t \rangle$  of hadrons produced in hadron inclusive reactions at low  $x$ , e.g.,  $x < 0.5$  and the strong  $x$ -dependence of  $\langle p_t \rangle(x)$  at  $x \rightarrow 1$ , see for example [18] and references therein.

Developing the approach of the previous section one can fit the quark distribution  $g(k_t)$  also in the statistical form (2), i.e.,

$$g(k_t) \sim \exp\left(-\frac{\epsilon_{kt}}{T}\right), \quad (8)$$

where  $\epsilon_{kt} = \sqrt{k_t^2 + m_q^2}$  is the transverse energy of quarks in proton and  $m_q$  is the quark mass. For the massless quarks (the current quarks)  $g(k_t)$  can be represented in the following form:

$$g(k_t) \sim \exp\left(-\frac{k_t}{T}\right) = \exp\left(-\frac{k_t}{\langle k_t \rangle}\right). \quad (9)$$

Applying here the same effective temperature  $T \simeq 120 - 150$  MeV, as for longitudinal momentum distribution, we get similar results on  $\langle k_t \rangle$ , which were obtained for valence current quarks (see e.g.[14] and Ref. therein). Actually, these values for  $\langle k_t \rangle$  of quarks in proton are used to get the inclusive transverse momentum spectra of hadrons produced in  $p-p$  collisions at not large  $p_t$ , which were obtained in [18]-[22] within the dual parton model (DPM) [19] or the quark-gluon string model (QGSM) [20].

Therefore, the transverse momentum distribution of partons in proton can also be described in the statistical form with the same value of the temperature  $T$  as the parton distributions in proton over the longitudinal momentum or over its fraction  $x$ .

Note that the longitudinal and transverse momentum dependences have been related in a model approach implying Lorentz (rotational) invariance [14, 23].

### 3. Inclusive hadron spectra in $N - N$ and $A - A$ collisions

The inclusive spectra of hadrons produced in  $N - N$  collisions at high energies in the central region are not described by the statistical Fermi-Dirac distribution given by Eq.(1) (see, e.g., the fit of experimental data in [24] and references therein). For example, the transverse momentum  $p_t$  spectra of all the hadrons produced in  $p - p$  collisions at high energies have a more complicated form than Eq.(1). However, the experimental data on these spectra at not large values of  $p_t$  can be described [18]-[22] within the QGSM using an exponential form similar to Eq.(9) for the quark distribution in proton on the internal transverse momentum  $k_t$ .

This manifests another application of the duality principle suggested in Section Ib.

Contrary to this, the experimental ratios of hadron yields and transverse mass spectra of hadrons produced in central heavy-ion collisions are described rather satisfactorily by the statistical (thermal) model, being its traditional field of application (see, for example [1, 25, 4], [26]-[28], and references therein). Note that the freeze-out temperature parameter obtained in this analysis at the zero chemical potential is also compatible with our estimation  $T \sim 150$  MeV.

### 4. Conclusion

We analyzed the parton distribution functions in a proton using the statistical model [6, 8, 9]. We suggest a duality principle which means a similarity of the thermal distributions of partons (quarks and gluons) and the dynamical description of these partons. This duality allowed us to find an *effective temperature*  $T \sim 120 - 150$  MeV for the massless quarks. These values for  $T$  are the same for the distributions of partons on the longitudinal and transverse momentum.

The inclusive spectra of hadrons produced in  $NN$  collisions at high energies are not described by the thermal statistical distribution. However, using the thermal statistical form for quark distribution as a function of the longitudinal momentum fraction  $x$  and the internal transverse momentum  $k_t$  one can get a satisfactory description of the experimental data on these spectra.

It happens that the freeze-out temperature in central heavy-ion collisions at zero chemical potential has a similar value which was estimated in this paper for the massless quarks in a free nucleon.

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